

Streamline flow	Turbulent flow
1) The smooth flow of a fluid, with velocity smaller than certain critical velocity (limiting value of velocity) is called streamline flow or laminar flow of a fluid.	1) The irregular and unsteady flow of a fluid when its velocity increases beyond critical velocity is called turbulent flow.
2) In a streamline flow, velocity of a fluid at a given point is always constant.	2) In a turbulent flow, the velocity of a fluid at any point does not remain constant.
3) Two streamlines can never intersect, i.e., they are always parallel.	3) In a turbulent flow, at some points, the fluid may have rotational motion which gives rise to eddies.
4) Streamline flow over a plane surface can be assumed to be divided into a number of plane layers. In a flow of liquid through a pipe of uniform cross sectional area, all the streamlines will be parallel to the axis of the tube.	4) A flow tube loses its order and particles move in random direction.

Critical Velocity and Reynolds Number:

$R_n = \frac{V_c \rho d}{\eta}$ where R_n = Reynolds number, V_c = critical velocity, η = coefficient of viscosity, ρ = density of fluid, d = diameter of tube

$R_n < 1000$, streamline flow
 $R_n > 2000$, turbulent fluid
 $1000 < R_n < 2000$, transition phase

Viscosity:

Viscosity is the property by virtue of which, the relative motion between different layers of the fluid experience a dragging force

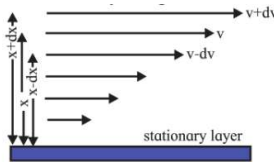
In liquids drag is due to molecular cohesion and in gasses its due to collisions

In streamline flow, Viscous drag is proportional to relative velocity.

According to Newtons law of viscosity,

$$f \propto A \frac{dv}{dx}$$

$$f = \eta A \frac{dv}{dx}$$

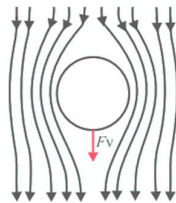


Where f = viscous force, A = area of the layer, dv/dx = velocity gradiet
 η = coefficient of viscosity and is defined as viscous force per unit area per unit velocity gradient. S.I. Unit Ns/m^2

Note: Fluid with no drag => where every layer is moving with same velocity e.g. liquid helium at 2K

Stokes Law:

The viscous force F_v acting on a small sphere falling through a viscous medium is directly proportional to the radius of the sphere (r), its relative velocity(v) through the fluid, and the coefficient of viscosity(η) of the fluid



$$F_v \propto \eta r v$$

$$F_v = 6\pi \eta r v$$

Terminal Velocity:

Consider a spherical body falling through a viscous fluid. The forces acting on it are

$mg = V \cdot \rho_s \cdot g = \frac{4}{3} \pi r^3 \rho_s g$ = its weight (downward)

$U = V \cdot \rho_L \cdot g = \frac{4}{3} \pi r^3 \rho_L g$ = upthrust (upwards)

F_v = viscous force (upwards) = $6\pi \eta r v$

Net downward force = $mg - U - F_v$

till the net force exists velocity in downward direction keeps increasing and hence F_v keeps increasing. There will be a moment when net force = 0 and then the body will sink with a constant velocity (Since net force=0).

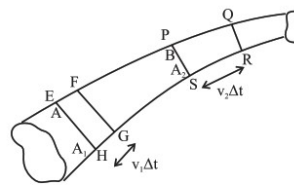
This constant velocity is called terminal velocity.

Then, $U + F_v = mg$

$$\frac{4}{3} \pi r^3 \rho_L g + 6\pi \eta r v = \frac{4}{3} \pi r^3 \rho_s g$$

$$\text{Thus, } v = \frac{2r^2 g (\rho_s - \rho_L)}{9\eta}$$

Law or Equation of continuity:



Let v_1 speed of fluid at A and v_2 be the speed of fluid at B
 A_1 is the area of cross-section at A and A_2 is the area of cross-section at B (where $A_2 < A_1$)

Mass of fluid entering A in time Δt = Mass of fluid exiting B in time Δt

$\rho \cdot (\text{volume at A in time } \Delta t) = \rho \cdot (\text{volume at B in time } \Delta t)$

$$\rho(A_1 v_1) = \rho(A_2 v_2)$$

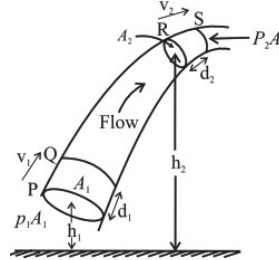
Thus, $A_1 v_1 = A_2 v_2$ OR $Av = \text{constant}$

Thus smaller the area, greater is the velocity (assuming negligible friction and incompressible fluid)

NOTE: Av is called rate of flow or volume rate of flow or volume flux

ρAv is called mass rate of flow or mass flux

Bernoulli Equation



Let P_1 be the pressure and v_1 be the velocity of the fluid at PQ and A_1 the area of cross-section. Thus the force at this end is $P_1 A_1$.

Let p_2 be the pressure and v_2 be the velocity of the fluid at RS and A_2 the area of cross-section. Thus the force at this end is $p_2 A_2$.

Let $h = h_2 - h_1$ be the difference in height.

Since $A_2 < A_1$, $v_2 > v_1$ (by principle of continuity), hence $d_2 > d_1$ where d_1 and d_2 is the distance covered in time Δt

Thus, work done in time $\Delta t = W = p_1 dV_1 - p_2 dV_2 = p_1 A_1 d_1 - p_2 A_2 d_2$

Note $dV_1 = dV_2 (= \text{Volume in time } \Delta t)$. Thus, $W = (P_1 - P_2) dV$ (i)

But $W = \Delta KE + \Delta PE$ where ΔKE and ΔPE are the change in time Δt

$\Delta KE = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 = \frac{1}{2} (\rho dV_1) v_2^2 - \frac{1}{2} (\rho dV_2) v_1^2 = \frac{1}{2} \rho dV (v_2^2 - v_1^2)$..(ii)

$\Delta PE = mgh_2 - mgh_1 = \rho dV g (h_2 - h_1)$ (iii)

Using (i), (ii) and (iii)

$$(P_1 - P_2) dV = \frac{1}{2} \rho dV (v_2^2 - v_1^2) + \rho dV g (h_2 - h_1)$$

Work done per unit volume of fluid by surrounding fluid is equal to sum of the change in KE & PE per unit volume that occurs during the flow.

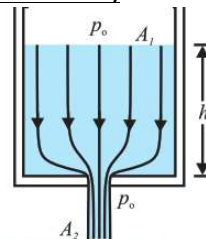
Thus, $P_1 + \frac{1}{2} \rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho gh_2$

Thus, $P + \frac{1}{2} \rho v^2 + \rho gh = \text{constant}$

Work done per unit volume of a fluid by the surrounding fluid is equal to the sum of the change in KE and PE per unit volume that occurs during the flow.

Applications:

• Efflux Velocity



Consider a large open tank with Area of cross-section A_1 and atmospheric pressure p_0 . Let there be a small orifice of area A_2 and liquid of density ρ leaving the orifice with velocity v_2 .

By law of continuity, $A_1 v_1 = A_2 v_2$

$$\text{Thus, } v_1 = \frac{A_2}{A_1} v_2$$

By Bernoulli Equation

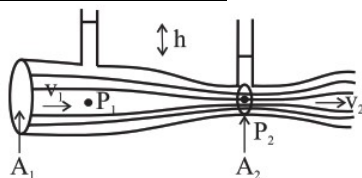
$$P_1 + \frac{1}{2} \rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho gh_2$$

$$p_0 + \frac{1}{2} \rho \left(\frac{A_2 v_2}{A_1} \right)^2 + \rho gh = p_0 + \frac{1}{2} \rho v_2^2$$

$$\text{Thus, } v_2 = \sqrt{\frac{2gh}{1 - \left(\frac{A_2}{A_1}\right)^2}}. \text{ Since } A_2 \ll A_1, \quad v_2 = \sqrt{2gh}$$

NOTE: The efflux velocity is numerically same as the velocity of a point mass dropped from a height h .

Ventury Meter/Ventury Tube



A ventury meter is used to measure the speed flow of a fluid.

ρ = density of fluid

A_1 and A_2 = area of cross section of the tube

v_1 and v_2 = velocity of the fluid

By law of continuity, since $A_1 > A_2$ thus $v_1 < v_2$

From Bernoulli's principle,

$$P_1 + \frac{1}{2} \rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho gh_2$$

$$\text{Thus, } p_1 - p_2 = \frac{1}{2} \rho (v_2^2 - v_1^2)$$

This difference in pressure creates a height difference in the tube

$$\text{Thus, } h\rho g = \frac{1}{2} \rho (v_2^2 - v_1^2)$$

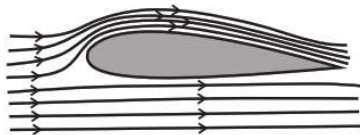
$$\text{Since } A_1 v_1 = A_2 v_2$$

$$h\rho g = \frac{1}{2} \rho \left(\left(\frac{A_1 v_1}{A_2} \right)^2 - v_1^2 \right)$$

$$2gh = v_1^2 \left(\left(\frac{A_1}{A_2} \right)^2 - 1 \right)$$

$$v_1 = \sqrt{\frac{2gh}{\left(\frac{A_1}{A_2} \right)^2 - 1}}$$

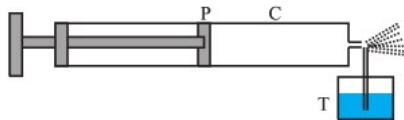
Lifitng of an Aeroplane



Due to the shape of the cross section of the wing the air is more crowded on top of the wing than below. Hence the speed of the wind is more

above. Hence by Bernoulli's principle the pressure on top is less than that below, due to which the plane gets an uplift (upward force called dynamic lift).

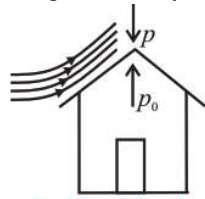
Atomizer



Tube T is dipped in the liquid. Air at high speeds is blown over the tip of this tube with the help of a piston P. This high

speed air creates a low pressure over the tube, due to which the liquid rises in it and is blow off in very small droplets with the expelled air. e.g. carburetor of an automobile engine, paint-gun, scent-spray, insect-sprayer.

Blowing off of roofs by stormy winds



The high speed winds creates a low pressure (p) above the roof (in accordance to Bernoulli's principle). Inside the room its still atmospheric pressure (p_0). This difference in pressure will cause the roof to be lifted up and then blown away by the wind

Ideal Fluid:

It is incompressible (density is constant)

Its flow is irrotational (no turbulence in flow)

It is nonviscous (no internal friction)

Has Steady flow (velocity at each point is constant in time)

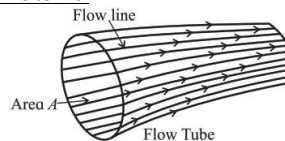
Properties of Fluid:

They do not oppose deformation completely(they get permantly deformed)

They have the ability to flow

They have ability to take shape of container

Some terms:



Steady Flow : Measurable property like pressure or velocity of a fluid at a given point is constant over time.

Flow Line : Path of an individual particle in a moving fluid

StreamLine : A curve whose tangent at any point in the flow is in the direction of the velocity of the flow at that point. Streamlines and flow lines are identical for a steady flow.

Flow tube : It is an imaginary bundle of flow lines bound by an imaginary wall. For steay flow, the fluid cannot cross the walls of a flow tube. Fluids in adjacent flow tubes cannot mix.

Laminar Flow or Streamline Flow : It is a steady flow in which adjacent layers of a fluid move smoothly over each other.(Reynolds Number < 1000)

Turbulent flow : It is a flow at a very high flow rate. There is not steady flow and the flow pattern keeps changing.(Reynolds Number > 2000)

Hydrostatics: The branch of physics which deals with properties of fluids at rest

